

**MATHEMATICS
STANDARD LEVEL
PAPER 2**

Thursday 8 May 2008 (morning)

Candidate session number

1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATE

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section a

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

In a school with 125 girls, each student is tested to see how many sit-up exercises (sit-ups) she can do in one minute. The results are given in the table below.

n umber of sit-ups	n umber of students	c umulative number of students
15	11	11
16	21	32
17	33	p
18	q	99
19	18	117
20	8	125

(Question 1 continued)

(a) (i) Write down the value of p .

(ii) Find the value of q .

[3 marks]

(b) Find the median number of sit-ups.

[2 marks]

(c) Find the mean number of sit-ups.

[2 marks]

2. [Maximum mark: 6]

The diagram below shows triangle PQR. The length of [PQ] is 7 cm, the length of [PR] is 10 cm, and $\angle PQR$ is 75° .

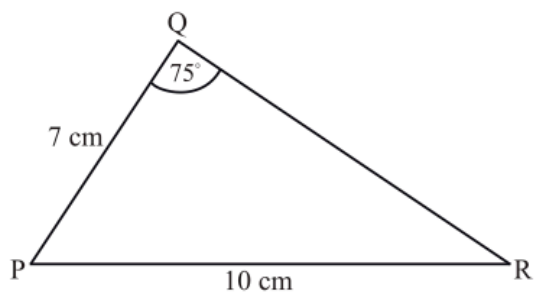
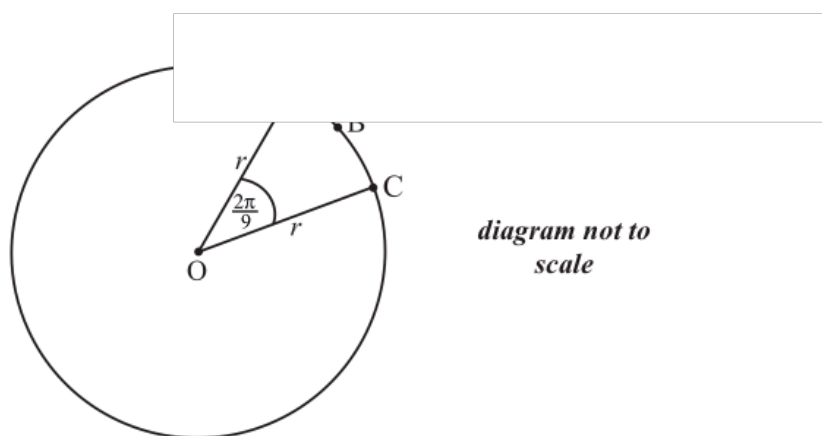


diagram not to scale

- (a) Find $\angle PRQ$. [3 marks]
- (b) Find the area of triangle PQR. [3 marks]

3. [Maximum mark: 6]

The diagram below shows a circle centre O , with radius r . The length of arc ABC is 3π cm and $\angle AOC = \frac{2\pi}{9}$.



(a) Find the value of r . [2 marks]

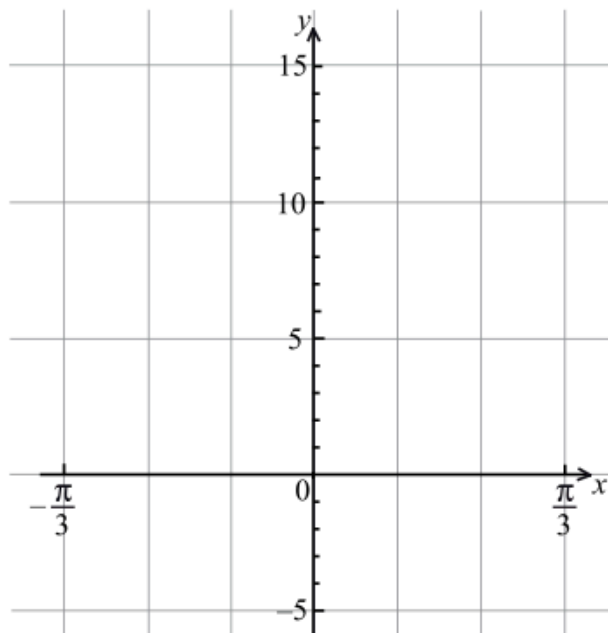
(b) Find the perimeter of sector $OABC$. [2 marks]

(c) Find the area of sector $OABC$. [2 marks]

4. [Maximum mark: 6]

Let $f(x) = 4 \tan^2 x - 4 \sin x$, $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$.

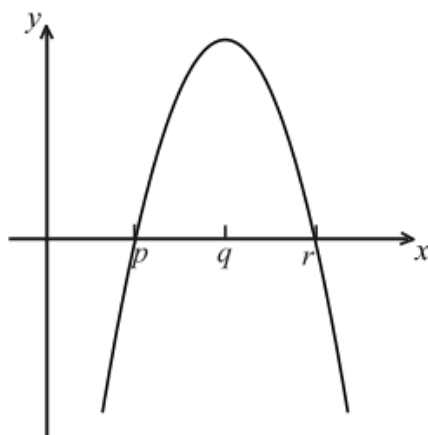
(a) On the grid below, sketch the graph of $y = f(x)$.



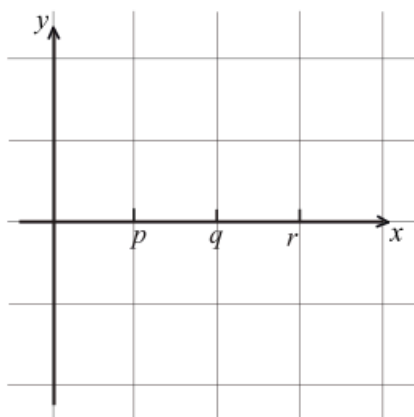
(b) Solve the equation $f(x) = 1$.

5. [Maximum mark: 6]

The diagram below shows part of the graph of the gradient function, $y = f'(x)$.



- (a) On the grid below, sketch a graph of $y = f''(x)$, clearly indicating the x -intercept. [2 marks]



- (b) Complete the table, for the graph of $y = f(x)$. [2 marks]

	x -coordinate
(i) Maximum point on f	
(ii) Inflexion point on f	

- (c) Justify your answer to part (b)(ii). [2 marks]

6. *[Maximum mark: 7]*

A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches.

- (a) Find the mean number of defective switches in the sample. *[2 marks]*
- (b) Find the probability that there are exactly six defective switches in the sample. *[2 marks]*
- (c) Find the probability that there is at least one defective switch in the sample. *[3 marks]*

7. [Maximum mark: 7]

Let $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. The vector $\mathbf{v} + p\mathbf{w}$ is perpendicular to \mathbf{w} . Find the value of p .

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 13]

A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

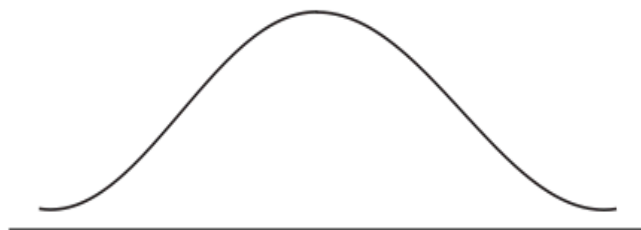
(i) weighs less than 8 g;

(ii) weighs between 6 g and 8 g.

[4 marks]

(b) Five percent of the biscuits in the box weigh less than d grams.

(i) Copy and complete the following
this information, by indicating d ,



(ii) Find the value of d .

[5 marks]

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20 % of the biscuits in this second box weigh less than 5 g.

Find the value of μ .

[4 marks]

9. [Maximum mark: 18]

The point O has coordinates $(0, 0, 0)$, point A has coordinates $(1, -2, 3)$ and point B has coordinates $(-3, 4, 2)$.

(a) (i) Show that $\vec{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find \hat{BAO} .

[8 marks]

(b) The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1 .

[2 marks]

(c) The line L_2 passes through A and is parallel to \vec{OB} .

(i) Find a vector equation for L_2 , giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

(ii) Point C $(k, -k, 5)$ is on L_2 . Find the coordinates of C.

[6 marks]

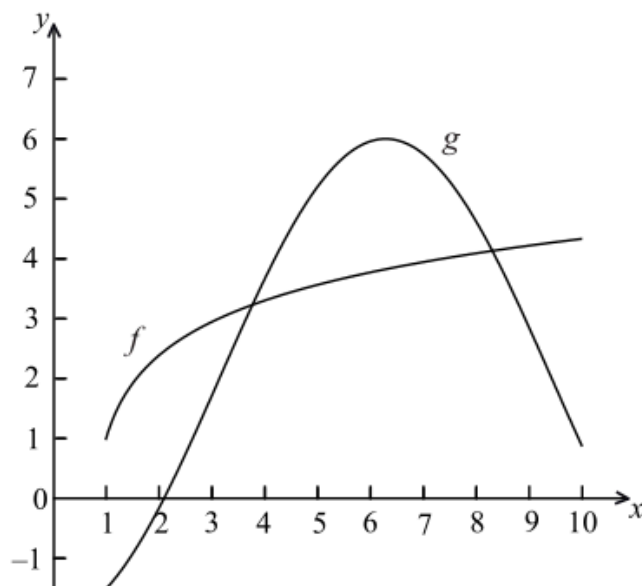
(d) The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, and passes through the point C.

Find the value of p at C.

[2 marks]

10. [Maximum mark: 14]

The following diagram shows the graphs of $f(x) = \ln(3x-2)+1$ and $g(x) = -4\cos(0.5x)+2$ for $1 \leq x \leq 10$.



(a) Let A be the area of the region enclosed by the curves of f and g .

(i) Find an expression for A .

(ii) Calculate the value of A .

[6 marks]

(b) (i) Find $f'(x)$.

(ii) Find $g'(x)$.

[4 marks]

(c) There are two values of x for which the gradient of f is equal to the gradient of g . Find both these values of x .

[4 marks]